ST.ANNE'S

COLLEGE OF ENGINEERING AND TECHNOLOGY

(An ISO 9001:2015 Certified Institution)

Anguchettypalayam, Panruti – 607106.

QUESTION BANK

PERIOD: DEC-18 – MAR-19 **BATCH**: 2015 – 2019

BRANCH: ECE **YEAR/SEM:** II/04

SUB CODE/NAME: MA8451 – PROBABILITY AND RANDOM PROCESSES

UNIT I – PROBABILITY AND RANDOM VARIABLES

PART – A

1. Define random variable.
2. X and Y are independent random variables with variances 2 and 3. Find the variance of 3X +4Y.
3. Let X be a R.V with E[X]=1 and E[X(X-1)]=4 . Find var X and Var(2-3X).
4. The number hardware failures of a computer system in a week of operations as the following pmf:

Number of failures:	0	1	2	3	4	5	6
Probability :	0.18	0.28	0.25	0.18	0.06	0.04	0.01

   Find the mean of the number of failures in a week

5. A continuous random variable X has the probability density function given by $f(x) = 3x^2, 0 \le x \le 1$. Find K such that P(X > K)= 0.5
6. A random variable X has the pdf f(x) given by $f(x) = \begin{cases} Cxe^{-x}, x > 0 \\ 0, x \le 0 \end{cases}$. Find the value of C and c.d.f of X.
7. The cumulative distribution function of a random variable X is F(x)= $[1-(1+x)e^{-x}], x > 0$. Find the probability density function of X.
8. Is the function defined as follows a density function? $f(x) = \begin{cases} 0, x < 2 \\ \frac{1}{18}(3+2x), 2 \le x \le 4 \\ 0, x > 4 \end{cases}$
9. Let X be a R.V with p.d.f given by $f(x) = \begin{cases} 2x, 0 < x < 1 \\ 0, otherwise \end{cases}$. Find the pdf of Y =(3X +1).
10. Find the cdf of a RV is given by $F(x) = \begin{cases} 0, x < 0 \\ \frac{x^2}{16}, 0 \le x \le 4 \\ 1, 4 < x \end{cases}$ and find P(X>1/X<3).
11. A continuous random variable X that can assume any value between x = 2 and x = 5 has a density function given by f(x) = K(1 + x). Find P[X<4].
12. The first four moments of a distribution about x = 4 are 1, 4, 10 and 45 respectively. Show that the mean is 5, variance is 3, $\mu_3 = 0$ and $\mu_4 = 26$.
13. Define moment generating function.

14. Write down the properties of moment generating function.

15. Find the moment generating function for the distribution where $f(x) = \begin{cases} \frac{2}{3}, x = 1 \\ \frac{1}{3}, x = 2 \\ 0, otherwise \end{cases}$.

16. For a binomial distribution mean is 6 and S.D is $\sqrt{2}$. Find the first two terms of the distribution.

17. Find the moment generating function of binomial distribution.

18. The mean of a binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution

19. If X is a Poisson variate such that P(X=2) = 9P(X=4) +90P(X=6),find the variance.

20. Write the MGF of geometric distribution.

21. One percent of jobs arriving at a computer system need to wait until weekends for scheduling , owing to core-size limitations. Find the probability that among a sample of 200 jobs there are no job that have to wait until weekends.

22. Show that for the uniform distribution $f(x) = \frac{1}{2a}, -a < x < a$ the moment generating function about origin is $\frac{\sinh at}{at}$.

23. If X is a Gaussian random variable with mean zero and variance σ^2 , find the probability density function of $Y = |X|$.

24. A random variable X has p.d.f $f(x) = \begin{cases} e^{-x}, x > 0 \\ 0, x < 0 \end{cases}$. Find the density function of $\frac{1}{x}$

25. State Memoryless property of exponential distribution.

26. The mean and variance of binomial distribution are **5** and **4**. Determine the distribution.

27. For a binomial distribution mean is **6** and S.D is $\sqrt{\mathbf{2}}$. Find the first of the distribution.

28. What are the limitations of Poisson distribution.

29. A random variable X is uniformly distributed between 3 and 15.Find mean and variance.

30. A continuous random variable X has a p.d.f given by $f(x) = \begin{cases} \frac{3}{4}(2x - x^2), 0 < x < 2 \\ 0 \quad , otherwise \end{cases}$ .Find $p(x > 1)$

31. Let X be the random variable which denotes the number of heads in three tosses of a fair coin. Determine the probability mass function of X.

32. If $f(x) = \begin{cases} ke^{-x}, x > 0 \\ 0, otherise \end{cases}$ is p.d.f of a random variable X ,then the value of K.

33. Find the mean and variance of the discrete random variable X with the p.m.f $p(x) = \begin{cases} \frac{1}{3}, x = 0 \\ \frac{2}{3}, x = 2 \end{cases}$

34. A random variable X has c.d.f $F(x) = \begin{cases} 0 \quad , x < 1 \\ \frac{1}{2}(x - 1), 1 \le x \le 3 \\ 0 \quad , x > 3 \end{cases}$. Find the p.d.f of X and the expected value of X.

PART – B

FIRST HALF (All are 8- marks)

(A) DISCRETE DISTRIBUTION:-

I- Binomial distribution

1. ***The moment generating function of a binomial distribution and find mean and variance (i.e)*** $p(x) = \begin{cases} nc_x p^x q^{n-x}, x = 0,1,2,3 \ldots\ldots \\ 0 \quad ,otherwise \end{cases}$
2. The probability of a bomb hitting a target is $\frac{1}{5}$. Two bombs are enough to destroy a bridge. If six bombs are aimed at the bridge, find the probability that the bridge is destroyed?
3. If 10% of the screws produced by an automatic machine are defective find the probability that out of 20 screws selected at random, there are
   (i) exactly 2 defective (ii) at most 3 defective (iii) at least 2 defective

   (iv) between 1 & 3 defective
4. the probability of a man hitting a target is $\frac{1}{4}$. If he fires 7- times (i) what is the probability of his hitting the target at least twice? (ii) How many times must he fire so that the probability of hitting the target at least once is greater than $\frac{2}{3}$.
5. A machine manufacturing screws is known to probability 5% defective in a random sample of 15 screws, what is the probability that there are (i) exactly 3 defective (ii) not more than 3 defectives.
6. Five fair coins are flipped. If the outcomes are assumed independent find the probability mass function of the number of heads obtained.

II- Poisson distribution

1. ***The moment generating function of a Poisson distribution and find mean and variance (i.e)*** $P(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, x = 0,1,2,3, \ldots\ldots\ldots \\ 0 \quad ,otherwise \end{cases}$
2. ***Derive the Poisson distribution as a limiting case of binomial distribution***.

3. If 3% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective.

4. The no.of monthly breakdown of a computer is a random variable having a Poisson distribution with mean equal to 1.8, Find the probability that this computer will function a month (i) without a breakdown (ii) with only one breakdown (iii) with at least one breakdown .

5. If X is a Poisson variate such that $p(x=1)=\frac{3}{10}$ and $p(x=2)=\frac{1}{5}$, find $p(x=0)\ and\ \ p(x=3)$.

6. If X is a Poisson variate such that $P\ (X=2)=9\ P(X=4)+90\ P(X=6)$ find the mean and variance.

III- Geometric distribution

1. The moment generating function of a Geometric distribution and find mean and variance (i.e) $p(x)=q^{x-1}P\ \ ,x=1,2,3,\dots\dots\ \ where\ q=1-p$.

2. ***State and prove the memory less property of the geometric distribution***.

3. Suppose that a trainee soldier shoots a target in a independent fashion. If the probability that the target is shot on any one shot is 0.8. (i) what is the the probability that the target would be hit on 6^{th} attempt (ii) what is probability that it takes him less than 5 shots (iii) what is probability it takes him even no. of shots.

4. If the probability that a target is destroyed on any one shot is 0.5, what is the probability that it would be destroyed on 6^{th} attempt?

5. If the probability that an applicant for a driver license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (i) on the 4^{th} trial. (ii) in fewer than 4 trials.

IV- Uniform distribution

1. The moment generating function of a uniform distribution and find mean and variance .

2. Buses arrive at a specified bus stop at 15-min intervals starting at 7 a.m that is 7a.m ,7.15am ,7.30am,…..etc. If a passenger arrives at the bus stop at a random time which is uniformly distributed between 7am and 7.30 am.Find the probability that he waits (i) less than 5 min (ii) atleast 12 min for bus.

3. Subway trains on a certain run every half an hour between mid-night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least 20-minutes.

Problems on Discrete random variables:-

1. If the probability mass function of a random variable X is given by $P[X = x] = kx^3, x = 1,2,3,4,$ (i) find the value of K.(ii) $P\left[\left(\frac{1}{2} < X < \frac{5}{2}\right) / X > 1\right]$

(ii) Mean and variance.

2. A random variable X has the following probability function

$X=x_i$	0	1	2	3	4	5	6	7
$P(X=x_i)$	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) Find the value of K (ii) $P(X < 6)$ (iii)$P(X \geq 6)$ (iv) $P(1 \leq x \leq 5)$

SECOND HALF (All are 8- marks)

(B) Continuous DISTRIBUTION:-

I- Exponential distribution

1. *The moment generating function of a exponential distribution and find mean and variance. (i.e)* $P(x) = \begin{cases} \lambda e^{-\lambda x}, x \geq 0, \lambda > 0 \\ 0 \quad , otherwise \end{cases}$

2. *State and prove the memory less property of the exponential distribution.*

3. The time in hours required to repair a machine is exponentially distribution with perimeter $\lambda = \frac{1}{2}$. (i) what is probability that the repair time exceeds 2 hours. (ii)What is the conditional probability that repair takes at least 10 hours given that is duration exceeds 9 hours.

4. The length of time a person speaks over phone follows exponential distribution with mean 6. What is the probability that the person will take for (i) more than 8-minutes (ii) between 4 and 8 minutes.

II- Gamma distribution

1. The moment generating function of a exponential distribution and find mean and variance.

2. In a certain city the daily consumption of electric power in million of kilowalt hours can be treated as a random variable having an Erlang distribution with parameters $\left(\frac{1}{2}, 3\right)$. If the power plant of this city has a daily capacity of 12 millions kilowalt hours, what is the probability that this power supply will be inadequate on any given day?
3. Suppose that telephone calls arriving at a particular switchboard follow a poison process with an average of 5 calls coming per minute. What is the probability that up to a minute will unit-2 calls have come into the switchboard?

III- <u>Normal distribution</u>

1. The moment generating functions of a normal distribution and find mean and variance.
2. In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and variance.
3. In a distribution exactly normal 75 of the items are under 35 and 39% are under 63. What are the mean and S.D of the distribution.
4. The peak temperature T, as measured in degrees Fahrenheit on a particularday is the Gaussian (85,10) random variables. What is $P(T > 100)$, $P(T < 60)$, $P(70 < T < 100)$.
5. An electrical firm manufactures light bulbs that have a life before burn out that is normally distributed with mean equal to 800 hrs and a standard deviation of 40 hrs. Find the (i) the probability that a bulb more than 834 hrs. (ii) the probability that bulbs burns between 778 and 834 hrs.

(C) <u>Problems on continuous random variables:-</u>

1. A continuous random variable X that can assume any value between X=2 and X=5 has a probability density function given by $f(x) = k(1 + x)$. Find $P(X < 4)$.

2. A continuous random variable X has pdf $f(x) = \begin{cases} \frac{k}{1+x^2}, -\infty < x < \infty \\ 0 \quad , otherwise \end{cases}$

Find (i) the value of K. (ii) $P(X > 0)$ (iii) distribution function of X.

3. If X is a continuous R.V's whose pdf is given by $f(x) = \begin{cases} c[4x - 2x^2], 0 < x < 2 \\ 0 \quad , otherwise \end{cases}$

4. The probability distribution function of a R.V's is $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$.

   Find the cumulative distribution function.

5. The density function of a R.V's X is given by $(x) = Kx(2 - x), 0 \leq x \leq 2$.
   Find the mean and variance.

6. The c.d.f of a R.V's X is $F(x) = 1 - (1 + x), x \geq 0.$ Find the p.d.f of X, mean and variance.

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UNIT-2 TWO-DIMENSIONAL RANDOM VARIABLES

PART – A

1. If two random variables X and Y have probability density function (PPF) $f(x,y) = ke^{-(2x+y)}$ for x ,y >0,evaluate k.
2. Define joint probability distribution of two random variables X and Y and state its properties.
3. If the point pdf of (X,Y) is given by $f(x,y) = e^{-(x+y)} x \geq 0, y \geq 0$ find E[XY].
4. If X and Y have joint pdf $f(x,y) = \begin{cases} x+y, 0 < x < 1, 0 < y < 1 \\ 0, otherwise \end{cases}$, check whether X and Y are ndependent.
5. Find the marginal density functions of X and Y if $f(x,y) = \frac{2}{5}(2x+5y), 0 \leq x \leq 1, 0 \leq y \leq 1$.
6. If the function $f(x,y) = c(1-x)(1-y), 0 < x < 1, 0 < y < 1, y < 1$ to be a density function,find the value of c.
7. Let X and Y be continuous RVs with J.p.d.f $f(x,y) = \begin{cases} 2xy + \frac{3}{2}y^2, 0 < x < 1, 0 < y < 1 \\ 0, otherwise \end{cases}$ .Find P(X +Y<1).
8. The regression lines between twqo random variables X and Y is given by 3X +Y =10 and 3X +4Y =12. Find the co-efficient of correlation between X and Y.
9. If X and Y are random variables such that Y = aX +b where a and b are real constants, show that the correlation co-efficient r(X,Y) between that has magnitude one.
10. If Y = -2X +3 , find the cov(X,Y).
11. Let (X,Y) be a two dimensional random variable. Define covariance of (X,Y). If X and Y are independent , what will be the covariance of (X,Y).
12. Prove that Cov(aX+bY)=abCov(X ,Y).
13. Find the angle between the two lines of regression.
14. The regression equations of X on Y and Y on X are respectively 5x –y =22 and 64x -45y =24. Find the means of X and Y.
15. The tamgent of the angle between the lines of regression Y on X and X on Y is 0.6 and $\sigma_x = \frac{1}{2}\sigma_y$. Find the correlation coefficient.
16. Distinguish between correlation and regression.
17. State the central limit theorem for independenc and identically distributed random variables.

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18. 1. The two regression equations of two random variables X and Y are $4x - 5y + 33 = 0$ and $20x - 9y = 107$. find mean values of X and Y.

19. The angle between the two lines of regression.

20. If X and y are independent random variables with variance 2 and 3, then find the variance of $3x + 4y$.

21. The lines of regression in a bivariate distribution are $x + 9y = 7$ and $y + 4x = \frac{49}{3}$. find the coefficient of correlation.

22. If the joint P.d.f of (x, y) is $f(x, y) = \begin{cases} \frac{1}{4}, & 0 \le x, y \le 2 \\ 0, & \text{otherwise} \end{cases}$ Find $P[x + y \le 1]$

23. If two random variable X and Y have P.d.f $f(x, y) = Ke^{-(2x+y)}$, for $x, y \ge 0$. Find the value of K.

24. Find K if the joint p.d.f of a bivariate random variable is given by $f(x, y) = \begin{cases} k(1 - x)(1 - y), & 0 < (x, y) < 1 \\ 0, & \text{otherwise} \end{cases}$

25. 8. If the joint p.d.f of (x, y) is $f(x, y) = \begin{cases} e^{-(x+y)}, & x>0, y>0 \\ 0, & \text{otherwise} \end{cases}$ Check whether X and Y are independent.

26. The joint p.m.f of a two dimensional random variable (x,y) is given by $\mathbf{P(x, y) = K(2x + y),\ x = 1, 2}$ and $\mathbf{y = 1, 2}$, where K is a constant. Find the value of K.

PART – B

FIRST HALF (All are 8- marks)

I- Problems on discrete random variable

1. The joint PMF of two random variables X and Y is given by
   $P(x, y) = \begin{cases} k(2x + y), & x = 1,2; y = 1,2 \\ 0, & \text{otherwise} \end{cases}$, where K is a constant.
   (i) Find the K. (ii) Find the marginal PMF of X and Y.
2. The joint probability mass function of (x,y) is given by $P(x, y) = \frac{1}{72}(2x + 3y)$ $x = 0,1,2$ and $y = 1,2,3$. Find all the marginal and conditional probability distribution of X and Y.

3. The joint probability mass function of (x,y) is given by $P(x, y) = K(2x + 3y)$

$x = 0,1,2\,; y = 1,2,3$. (i) Find all the marginal and conditional probability distribution. Also find the probability distribution of $(x + y)$ *and* $P[x + y > 3]$

4. The joint distribution of X and Y is given by $f(x,y) = \frac{x+y}{21}\,, x = 1,2,3; y = 1,2$. Find the marginal distribution. Also find $E[xy]$.
5. Three balls are drawn at random without replacement from box containing 2- white , 3-red, 4-black balls. If X denote the no.of white balls and Y denote the no.of red balls drawn.Find the joint probability distribution of (X,Y)

II- Problems on continuous random variable

1. The joint PDF of (x,y) is $f(x,y) = e^{-(x+y)}, x, y \geq 0$. Are X and Y independent?
2. If the joint probability distribution function of a two dimensional random variable (x,y) is given by $f(x,y) = \begin{cases} (1-e^x)(1-e^y), & x > 0, y > 0 \\ 0 & ,\text{otherwise} \end{cases}$. (i) Find the marginal densities of X and Y. Are X and Y independent? (ii)$P[1 < x < 3\,, 1 < y < 2]$.
3. Given the joint density function of X and y as $f(x,y) = \begin{cases} \frac{1}{2}\, xe^{-y}; 0 < x < 2\,, y > 0 \\ 0 \quad ,elsewhere \end{cases}$. Find the distribution X+Y.
4. The joint PDF of the random variables (x,y) is given by $f(x,y) = k\, xy\, e^{-(x^2+y^2)}, x > 0, y > 0$. Find the value of K and also Prove that x and Y are independent.
5. If X and Y are two random variable having joint density function $f(x,y) = \begin{cases} \frac{1}{8}(6 - x - y), 0 < x < 2\,, 2 < y < 4 \\ 0 \quad ,\ otherwise \end{cases}$ .Find (i) $P[x < 1 \cap y < 3]$ (ii) $P[x + y < 3]$ (iii) $P[x < 1\,/ y < 3]$.
6. Suppose the point probability density function is given by $f(x,y) = \begin{cases} \frac{6}{5}\,(x+y^2\,); 0 < x < 1\,, 0 < y < 1 \\ 0 \quad ,elsewhere \end{cases}$. Obtain the marginal P.d.f of X and Y. Hence find $P\left[\frac{1}{4} \leq y \leq \frac{1}{2}\right]$
7. Given $f_{xy}(x,y) = c\, x(x - y), 0 < x < 2; -x < y < x,$ (i) Evaluate C (ii) Find $f_x(x)$ and $f_y(y)$ (iii) $f_{\frac{x}{y}}\left(\frac{x}{y}\right)$.

III- Problems on correlation and covariance

1. The joint PDF of a random variable (x,y) is $f(x,y) = 25e^{-5y}, 0 < x < 0.2, y > 0$. Find the covariance of x and Y.
2. Two random variables X and Y have the following joint p.d.f

$$f(x,y) = \begin{cases} 2 - x - y\,; 0 < x < 1\,, 0 < y < 1 \\ 0 \qquad ,elsewhere \end{cases}$$. (i) Find the $var(x) and\ var\ (y)$

(ii) The covariance between x and y. Also find ρ_{xy}.

3. If X and Y be discrete R.V's with p.d.f $f(x,y) = \frac{x+y}{21}\,, x = 1,2,3\,; y = 1,2$.
   (i) Find the mean and variance of X and Y, (ii) $cov(x,y)$ (iii) $r(x,y)$.

4. Two random variables X and Y have the following joint p.d.f

$$f(x,y) = \begin{cases} x + y\,;\ 0 < x < 1\,, 0 < y < 1 \\ 0 \qquad ,elsewhere \end{cases}$$. (i) Obtain the correlation co-efficient between X and y. (ii) Check whether X and Y are independent.

5. Find the coefficient of correlation between X and Y from the data given below.

X :	65	66	67	67	68	69	70	72
Y :	67	68	65	68	72	72	69	71

6. Find the coefficient of correlation between industrial production and export using the following data.

Production (x):	55	56	58	59	60	60	62
Export (y) :	35	38	37	39	44	43	44

SECOND HALF (All are 8- marks)

I- Problems on regression line

1. Two random variables X and Y are related as $y = 4x + 9$. find the correlation coefficient between X and Y.
2. The two lines of regression are $8x - 10y + 66\,;\ 40x - 18y - 214 = 0$. The

variance of X is 9. Find the mean values of X and Y. Also find the coefficient of correlation between the variables X and Y and find the variance of Y.

3. The two lines of regression are $8x - 10y + 66 \; and \;\; 40x - 18y - 214 = 0$. Find the mean values of X and Y. Also find the coefficient of correlation between the variables X and Y.

4. The regression equations are $3x + 2y = 26 \; and \; 6x + y = 31$. Find the correlation coefficient between x and y.

5. The equation of two regression lines are $3x + 12y = 19 \; and \; 3x + 9y = 46$. Find $\bar{x}$, $\bar{y}$ and the correlation coefficient between x and y.

II- Transformation of random variables

1. If X and Y are independent random variables with p.d.f $e^{-x}, x \geq 0$; $e^{-y}, y \geq 0$ respectively. Find the density function of $U = \frac{X}{X+Y} \; and \; V = X + Y$.

Are U &V independent.

2. The joint P.d.f of X and Y is given by $f(x, y) = e^{-(x+y)}, x \geq 0, y \geq 0$.

Find the probability density function of $U = \frac{X+Y}{2}$

3. If X and Y are independent exponential distributions with parameter 1 then find the P.d.f of $U = X - Y$.

4. Let (X, y) be a two-dimensional non-negative continuous random variable having the joint density $f(x, y) = \begin{cases} 4xy\, e^{-(x^2+y^2)} & , x \geq 0, y \geq 0 \\ 0 & , elsewhere \end{cases}$.

find the density of $U = \sqrt{x^2 + y^2}$

5. If the p.d.f of a two dimensional R.V (x,y) is given by $f(x, y) = x + y$,$0 \leq (x, y) \leq 1$. Find the p.d.f of $U = XY$.

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UNIT-3 RANDOM PROCESSES

PART – A

1. Define Sine-Wave Process.
2. State the four types of stochastic processes.
3. Prove that a first order stationary has a constant mean.
4. Define a stationary process (or)strictly stationary process (or)strict sense stationary process.
5. Consider the random process $X(t)=\cos(\omega_0 t+\theta)$, where θ is uniformly distributed in the interval $-\pi$ to π . Check whether X(t) is stationary or not?
6. When is a random process said to be ergodic.
7. Consider the Markov chain with tpm: $\begin{pmatrix} 0.4 & 0.6 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ is it irreducible? If not find the class. Find the nature of the states.
8. Define Markov chain and one-step transition probability.
9. State Chapman- Kolmogorow theorem.
10. What is a Markov process?
11. If the transition probability matrix of a Markov chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$, find the limiting distribution of the chain.
12. State any two properties of a Poisson process.
13. Prove that the difference of two independent poisson processes is not a poisson process.
14. If patients arrive at a clinic according to poisson process with mean rate of 2 per minute. Find the probability that during a 1-minute interval , no patients arrives.
15. The probability that a person is suffering from cancer is 0.001. Find the probability that out of 4000 persons (a) Exactly 4 suffer because of cancer, (b) more than 3 persons will suffer from the disease.
16. Define Stationary process.
17. Define Strictly stationary processes [sss]
18. Define Wide sense stationary processes [wss]

19. Define Ergodic random process.

20. What is meant by one-step transition probability?

21. What is random process? When do you say random process is a random variable?

22. Define Auto correlation.

23. Define power spectrum density.

24. State any two properties of cross correlation function.

25. State any two properties of an auto correlation function

26. State any two properties of cross power spectrum density

PART – B

FIRST HALF (All are 8- marks)

I- To find the wide sense stationary process

1. If $(t) = \sin(\omega t + y)$, where 'y' is uniformly distributed in $(0,2\pi)$.
Show that $x(t)$ is wide sense stationary process.

2. Show that the random process $x(t) = A\,cos(\omega t + \theta)$ is wide sense stationary**,** where $A\ and\ \omega$ are constants and θ is a uniformly distributed random in $(0,2\pi)$.

3. Show that the process $x(t) = Aco\,s\,\lambda t + Bsin\,\lambda t$ is wide sense stationary, where A and B are random variable if $E(A) = E(B) = 0$, $E(A^2) = E(B^2)$ and $E(AB) = 0$.
(or)
If $x(t) = Aco\,s\,\lambda t + Bsin\,\lambda t\,; t \geq 0$ is a random process where A and B are independent $N(0, \sigma^2)$ random variables examine the stationary of $x(t)$.

4. Let two random processes$\{x(t)\}\ and\ \{y(t)\}$ be defined as
$x(t) = Aco\,s\,\omega t + Bsin\,\omega t, y(t) = Bco\,s\,\omega t - Asin\,\omega t$,where A and B are random variables and ω is a constant. If $E(A) = E(B) = 0$, $E(A^2) = E(B^2)$ and $E(AB) = 0$.Prove that $\{x(t)\}\ and\ \{y(t)\}$ is jointly wide sense stationary.

5. If $x(t) = y\,co\,s\,t + z\,sin\,t, \forall\,t$ where y and z are independent binary random variables, each of which assumes the values -1 and 2 with probabilities $\frac{2}{3}\ and\ \frac{1}{3}$ respectively. Prove that $\{x(t)\}$ is wide sense stationary.

6. Show that the random process $x(t) = A\,co\,s\,t + B\,sin\,t\,, -\infty < t < \infty$ is a WSS process, where A and B are independent random variables each of which has -2 with probability $\frac{1}{3}$ and a value 1 with probability $\frac{2}{3}$.

7. The process $\{x(t)\}$ whose probability distribution under certain condition is given by

$$P\{x(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1,2,3, .. \\ \frac{at}{1+at} \quad , n = 0 \end{cases}$$ Show that it is not stationary.

8. Let the random process be $x(t) = \cos(t + \varphi)$ where φ is a random variable with density function $f(\varphi) = \frac{1}{\pi}$ $, -\frac{\pi}{2} < \varphi < \frac{\pi}{2}$ check whether the process is stationary or not.

9. Examine whether the Poisson process $\{x(t)\}$ given by the probability law $P\{x(t)\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$ $, n = 0,1,2, \dots \dots \dots \dots$ is not stationary.

II- Random telegraph process

1. Show that the random telegraph signal process is WSS

2. Show that the semi-random telegraph signal process is evolutionary (i.e) it is not a SSS and it is not a WSS process.

3. The auto correlation function of the random telegraph signal process is given by $R_{(\tau)} = \alpha^2 e^{-2\lambda|\tau|}$ .Determine the power density spectrum of the random telegraph signal.

4. Prove that a random telegraph signal process $y(t) = \alpha\, x(t)$ is a WSS process, when α is a random variable which is independent of $x(t)$, assume the values -1 and 1 with equal probability and $R_{xx}(t_1, t_2) = e^{-2\lambda(t_1 - t_2)}$

SECOND HALF (All are 8- marks)

III- Discrete parameter Markov process (Markov chain)

(a) To find the steady state distribution of the chain

1. An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals followed a highly distorted signal with no recognizable signal, whereas 20 out of 23 recognizable signals follow recognizable signals with no highly distorted signals between. Given that only highly distorted signals are not recognizable. Find the fraction of signals that are highly distorted.

2. A salesman's territory consists of three cities A,B and C, He never sells in the same city on successive days. If he sells in A, then the next day he sells in the city B. however, if he sells in either B or C, then the next day he twice as likely to sell in city A as in the other city. In the long run, how often does he sell in each of the cities.

(b) To find the probability distribution based on the initial distribution

1. The initial process of the Markov chain transition probability matrix is given by $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.2 & 0.7 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$ with initial probability $P_1{}^{(0)} = 0.4, P_2{}^{(0)} = 0.3, P_3{}^{(0)} = 0.3,$
Find $P_1{}^{(1)}, P_2{}^{(1)}, P_3{}^{(1)}$.

2. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drive one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drive to work if and only if a 6- appeared.
Find (i) the probability that the drives to work in the long run.
(ii) the probability that he takes a train on the 3rd day.

3. The transition probability matrix of the Markov chain $\{X_n\}$, n=1,2,3,…..having 3 states 1, 2 and 3 is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial is $P^{(0)} = [0.7 \quad 0.2 \quad 0.1]$
Find (i) $P(X_2 = 3)$ (ii) $P[X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2]$

4. The tpm of a Markov chain with three states 0,1,2 is $P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$ and the initial state distribution of the chain is $P(X_0 = i) = {}^1/_3$, $i = 0,1,2$. Find (i) $P(X_2 = 2)$
(iii) $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$.

5. The tpm of a Markov chain $\{X_n, n > 0\}$ have three states 0,1,2
$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$ with initial probability $P^{(0)} = [0.5 \quad 0.3 \quad 0.2]$
Find (i) $P(X_2 = 1)$ (ii) $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$.

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QUESTION BANK

PERIOD: DEC-18 – MAR-19 **BATCH**: 2017 – 2021

BRANCH: ECE **YEAR/SEM:** II/04

SUB CODE/NAME: MA8451 – PROBABILITY AND RANDOM PROCESSES

UNIT-4 CORRELATION AND SPECTRAL DENSITIES

PART – A

1. Define autocorrelation function and prove that for a WSS process {X(t)}, $R_{XX}(-\tau) = R_{XX}(\tau)$.
2. Stat any two properties of an auto correlation function.
3. $R_{XX}(\tau)$ is an even function of τ .
4. The power spectral density of a random process {X(t)} is given by $S_{XX}(\omega) = \begin{cases} \pi, |\omega| < 1 \\ 0, elsewhere \end{cases}$ .Find its autocorrelation function.
5. Find the variance of the stationary process {X(t)} whose ACF is given by $R(\tau) = 16 + \frac{9}{1+6\tau^2}$.
6. If the autocorrelation function of a stationary process is $R_{XX}(\tau) = 36 + \frac{4}{1+3\tau^2}$, find the mean and variance of the process.
7. Define Cross-correlation function and state any two of the properties.
8. What is meant by spectral analysis?
9. The power spectral density of a WSS process is given by $S(w) = \begin{cases} \frac{b}{a}(a-|w|); |w| \le a \\ 0; |w| > a \end{cases}$. Find the auto-correlation function of the process .
10. State any two uses of spectral density.
11. Given the power spectral density $S_{XX}(\omega) = \frac{1}{4+\omega^2}$, find the average power of the process.
12. Give an example of Cross spectral density.
13. Explain cross power spectrum.
14. Determine the cross-correlation function corresponding to the cross-power density spectrum
15. Check whether the following are valid autocorrelation function. (a) $2\sin \pi\tau (b) \frac{1}{1+4\tau^2}$
16. If $R(\tau) = e^{-1\lambda|\tau|}$ is the autocorrelation function of a random process X(t), obtain the spectral density of X(t).
17. State any two properties of cross correlation function.
18. State any two properties of an auto correlation function

19. State any two properties of cross power spectrum density.

20. Check whether the following are valid auto correlation function $R_{xx}(\tau) = \frac{1}{1+4\tau^2}$

21. Find the mean and variance of a stationary process whose $R_{xx}(\tau) = 16 + \frac{9}{1+6\tau^2}$

22. Check whether $S_{xx}(\omega) = \frac{2\omega^2+6}{8\omega^4+3\omega^2+4}$ can represent a valid power spectral density.

23. The power spectral density of a random process {x(t)} is

24. given by $S_{xx}(\omega) = \begin{cases} \pi, |\omega| < 1 \\ 0, elsewhere \end{cases}$ Find its auto correlation.

25. Find the mean square value of the random process, whose auto correlation is $\frac{A^2}{2}\cos\omega\tau$.

26. Find the power spectral density of the random process {x(t)}

27. Whose auto correlation is $R(\tau) = \begin{cases} -1, -3 < |\tau| < 3 \\ 0, elsewhere \end{cases}$

PART – B

FIRST HALF (All are 8- marks)

I- *Calculation of spectral density given the auto correlation*

1. The auto correlation of a stationary random process is given by $R_{xx}(\tau) = ae^{-b|\tau|}, b > 0$. find the spectral density function.
2. Calculate the power spectral density of a stationary random process for which the auto correlation is $R_{xx}(\tau) = e^{-\alpha|\tau|}$.
3. The auto correlation function of the random telegraph signal process is given by $R_{xx}(\tau) = a^2 e^{-2r|\tau|}$. Determine the power density spectrum of the random telegraph signal.
4. The auto correlation of the random binary transmission is given by
$R_{xx}(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, |\tau| \le T \\ 0 \quad\quad, |\tau| > T \end{cases}$. Find the power spectrum.
5. Consider the Ergodic random process whose auto correlation is
$R_{xx}(\tau) = \begin{cases} 1 - |\tau|, |\tau| \le T \\ 0 \quad\quad, |\tau| > T \end{cases}$. Find its spectral density.
6. Find the power spectral density of a WSS process with auto correlation function is $R_{xx}(\tau) = e^{\frac{-a^2\tau^2}{2}}$.

7. Find the power spectral density of a WSS process with auto correlation function is $R_{xx}(\tau) = e^{-\alpha\tau^2}, \alpha > 0$.

8. If $y(t) = X(t+a) - X(t-a)$, prove that $R_{xx}(\tau) = 2R_{xx}(\tau) - R_{xx}(\tau + 2a) - R_{xx}(\tau - 2a)$. Hence prove that $S_{yy}(\omega) = 4sin^2(a\omega).S_{xx}(\omega)$

II- ***Calculation of auto correlation given the power spectral density***

1. If the power spectral density of a WSS process is given by

$$s_{xx}(\omega) = \begin{cases} \frac{b}{a}(a - |\omega|), |\omega| \le a \\ 0 \qquad\qquad , |\omega| > a \end{cases}$$. Find the auto correlation function.

2. Th***e*** power spectrum of WSS process $\{X(t)\}$ is given by $s_{xx}(\omega) = \frac{1}{(1+\omega^2)^2}$.
Find the auto correlation function and hence the average power.

3. Given the power spectral density $s_{xx}(\omega) = \frac{\omega^2+9}{\omega^4+5\omega^2+4}$. Find the mean square value of the process.

4. Find the mean square value of the processes whose power spectral density is given below $s_{xx}(\omega) = \frac{1}{\omega^4+10\omega^2+9}$

SECOND HALF (All are 8- marks)

III- ***Calculation the cross correlation given Cross power spectral density function***

1. The cross power spectrum of real random processes $\{X(t)\}$ and $\{Y(t)\}$ is given by $s_{xy}(\omega) = \begin{cases} a + jb\omega, |\omega| \le 1 \\ 0 \quad , elsewhere \end{cases}$. Find the cross correlation function.
2. Find the cross correlation of the 2-processes $x(t)$ and $y(t)$ whose cross-power spectrum is given by $s_{xy}(\omega) = \begin{cases} p + \frac{iq\omega}{B}, B \le |\omega| \le B \\ \quad 0 \quad , elsewhere \end{cases}$
3. The cross-power spectrum of real random processes $x(t)$ and $y(t)$ is given by $s_{xy}(\omega) = \begin{cases} a + \frac{jbw}{\omega}, |\omega| \le 1 \\ 0 \quad , elsewhere \end{cases}$. Find the cross- correlation.
4. Determine the cross correlation function corresponding to the cross-power density spectrum $S_{xy}(\omega) = \frac{8}{(\alpha+j\omega)^3}$, where $\alpha > 0$ is a constant.

IV- <u>*To find the Cross correlation*</u>

1. Two random processes $\{X(t)\}$ and $\{Y(t)\}$ are given by $(t) = A\cos(\omega t + \theta)$, $y(t) = A\sin(\omega t + \theta)$ where A and B are constants and θ is a uniform random variable over 0 to 2π. Find the cross correlation function.

2. If $X(t) = 3\cos(\omega t + \theta)$ and $y(t) = 2\sin(\omega t + \theta - \frac{\pi}{2})$ are two random processes where θ is a random variable with uniformly distributed in $(0\,, 2\pi)$.
Prove that $\sqrt{R_{xx}(0).R_{xx}(0)} \geq \left|R_{xy}(\tau)\right|$

3. If $x(t)$ and $y(t)$ are two random processes then $\left|R_{xy}(\tau)\right| \leq \sqrt{R_{xx}(0).R_{xx}(0)}$ where $R_{xx}(\tau)$ and $R_{yy}(\tau)$ are their respective auto correlation function.

V- <u>*To find the mean and variance*</u>

1. The auto correlation function for a stationary process X(t) is given by $R_{xx}(\tau) = 9 + 2e^{-|\tau|}$. Find the mean of the random variables $y = \int_0^2 x(t)dt$ and variance of X(t)

2. The random process X(t) is stationary with $E[x(t)] = 1$ and $R_{xx}(\tau) = 1 + e^{-2|\tau|}$ Find the mean and variance of $S = \int_0^1 x(t)dt$.

3. A stationary random process X(t) with mean 2 has the auto correlation function $R_{xx}(\tau) = 4 + e^{\frac{-|\tau|}{10}}$. Find the mean and variance of $y = \int_0^1 x(t)dt$.

QUESTION BANK

PERIOD: DEC-18 – MAR-19 **BATCH**: 2017 – 2021

BRANCH: ECE **YEAR/SEM:** II/04

SUB CODE/NAME: MA8451 – PROBABILITY AND RANDOM PROCESSES

UNIT-5 LINEAR SYSTEMS WITH RANDOM INPUTS

PART – A

1. Describe a linear system.
2. Define a system. When is it called linear system?
3. State the properties of a linear filter.
4. Describe a linear system with an random input.
5. Give an example for a linear system.
6. Define Time invariance.
7. State Causality.
8. State stable.
9. Define White Noise (or) Gaussian Noise.
10. Define Thermal Noise.
11. Define Band-Limited White Noise.
12. Define Filters.
13. Find the auto correlation function of Gaussian white Noise.
14. State auto correlation function of Gaussian white noise.
15. Find the system transfer function , if a linear time invariant system has an impulse function.
16. A white signal with PSD $\frac{\eta}{2}$ is applied to an RC LPF. Find the auto correlation of the O/P signal of the filter.
17. Check whether the following matrix is stochastic ? $A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is it a regular matrix.
18. Check whether the system is time variant $y(t) = x(t)\cos\omega t$.
19. Check whether the system is linear $y(t) = 2\,x(t)$.
20. Define random telegraph process.
21. Define semi-random telegraph signal process.
22. Prove that random telegraph process $\{y(t)\}$ is a WSS.
23. State any two properties of a Poisson process.
24. Define Poisson process.

PART – B

FIRST HALF (All are 8- marks)

I- *Linear system and time invariant*

1. Examine whether the following system are linear.
   (i) $y(t) = \alpha\, x(t)$ (ii) $y(t) = t\, x(t)$ (iii) $y(t) = x^2 x(t)$

2. Examine whether the following system are time-invariant.
   (i) $y(t) = \alpha\, x(t)$ (ii) $y(t) = t\, x(t)$ (iii) $y(t) = x(t) - x(t-a)$

3. A WSS process $X(t)$ with $R_{xx}(\tau) = Ae^{-a|\tau|}$ where 'A' and 'a' are real positive constants is applied to the input of an linear time invariant system with
   $h(t) = e^{-bt}u(t)$ where 'b' is a real positive constant. Find the power spectrum density of the output of the system. Also find the auto correlation of the output $Y(t)$ of the system.
4. If $X(t)$ is the input voltage to a circuit and $y(t)$ is the output voltage ,then $\{X(t)\}$ is a stationary process with $\mu_x = 0$ and $R_{xx}(\tau) = e^{-2|\tau|}$. Find the mean of μ_y and power spectrum $S_{xx}(\omega)$ of the output of the system transfer function is given by $H(\omega) = \frac{1}{\omega+2i}$.
5. If $X(t)$ is the input voltage to a circuit and $y(t)$ is the output voltage ,then $\{X(t)\}$ is a stationary process with $\mu_x = 0$ and $R_{xx}(\tau) = e^{-\alpha|\tau|}$.
   Find μ_y , $Y_{yy}(\omega)$, if the power transfer function is $(\omega) = \frac{R}{R+iL\omega}$.
6. Show that $\{X(t)\}$ is a WSS process then the output $\{X(t)\}$ is a WSS process.

   (or)

   If the input to a time invariant stable linear system is a wide sense stationary process, prove that the output will also be a wide sense stationary process

7. A random $X(t)$ is the input to a linear system whose impulse function is $h(t) = 2e^{-t}, t \geq 0$. The auto correlation function of the process is $R_{xx}(\tau) = e^{-2|\tau|}$. Find the power spectral density of the output process $y(t)$.

8. If the output of the input $X(t)$ is defined as $y(t) = \frac{1}{T}\int_{t-T}^{T} X(s)ds$, prove that $X(t)\ and\ y(t)$ are related by means of convolution integral. Find the unit impulse response of the system.

9. A circuit has an impulse response given by $h(t) = \begin{cases} \frac{1}{T}, & 0 \le t \le T \\ 0 & ,elsewhere \end{cases}$.

Evaluate, $S_{yy}(\omega)\ in\ terms\ of\ S_{xx}(\omega)$.

10. A system has an impulse response $h(t) = e^{-\beta t}U(t)$, find the power spectral density of the output $Y(t)$ corresponding to the input $X(t)$.

<u>SECOND HALF (All are 8- marks)</u>

II- Auto correlation and cross correlation function of input and output.

1. Assume a random process $X(t)$ is given as input to a system with transfer $H(\omega) = 1\ for - \omega_0 < \omega < \omega_0$. If the auto correlation function of the input process is $\frac{N_0}{2}\ \delta(t)$. Find the auto correlation function of the output process.

2. Consider the white Gaussian noise zero mean and power spectral density $\frac{N_0}{2}$ applied to a low pass Rc filter where transfer function is $(f) = \frac{1}{1+2\pi jfRc}$. Find the spectral density and auto correlation function of the output process.

III- Properties of linear system.

1. If the input X(t) and its y(t) are related by $\mathrm{y(t)} = \int_{-\infty}^{\infty} \mathrm{h(u)x(t-u)du}$, then the system is a linear time invariant.

2. If $X(t)$ is a WSS process and if $\mathrm{y(t)} = \int_{-\infty}^{\infty} \mathrm{h(u)x(t-u)du}$, then $R_{yy}(\tau) = R_{xy}(\tau) * h(-\tau)$, where * denotes the convolution.

3. If the X(t) is a WSS process and if $\mathrm{y(t)} = \int_{-\infty}^{\infty} \mathrm{h(u)x(t-u)du}$, then $S_{xy}(\omega) = S_{xx}(\omega) * H(\omega)$

4. Show that $S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$, where $S_{xx}(\omega)\ and\ S_{yy}(\omega)$ are the power spectral density function of the input $X(t)$ and the output $Y(t)$ and $H(\omega)$ is the system transfer function.